Leveraging EU Green Investment Frameworks for Sustainable Portfolio Construction

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Abstract

The transition towards a sustainable economy requires investors to redirect capital towards activities aligned with European environmental objectives. In this paper, we introduce a portfolio optimization method integrating constraints derived from the European green taxonomy, notably through green CAPEX and green revenue indicators. Using data published by CAC 40 companies, we construct portfolios following the Markowitz approach, evaluating trade-offs between return, risk, diversification, and environmental alignment. The adopted approach enables exploration of possible compromises between financial performance and sustainability, considering both already-aligned activities and transitional investment efforts. Our results demonstrate that integrating ESG constraints can alter the efficient frontier, but that reasonable levels of alignment can be achieved without reducing the Sharpe ratio or diversification.

Keywords — EU Taxonomy, Portfolio optimization, Transition risk, European ESG regulation, ESG, Sustainable Finance

Summary

Introduction

Financial institutions and insurance companies are now required to publish their green asset ratios since 2024 in accordance with the EU Taxonomy Regulation (EU 2020/852), a cornerstone of the European sustainable finance framework. For banks, this concerns the *Green Asset Ratio* (GAR), which measures the share of bank assets that are aligned with the EU Taxonomy, including loans and other on-balance-sheet exposures. Insurers, on the other hand, must report the *Green Investment Ratio* (GIR), which captures the proportion of their investment portfolios that are aligned with the taxonomy, reflecting the sustainable share of assets. In Nexialog (2024), a comparative analysis of the sustainability practices of life insurers, as reflected in their LEC29 (Loi Énergie Climat) reports, shows that alignment ratios range between 0.5% and 6% in terms of turnover, and between 0.5% and 9% in terms of investment expenditures. These ratios are calculated following the classification defined by the European taxonomy, which provides a precise regulatory framework to define what constitutes a sustainable activity, relying on technical and/or qualitative criteria.

The use of these indicators in a portfolio optimization approach differs from earlier methods by its anchoring in a recognized European regulatory standard. It thus offers greater methodological robustness and lower exposure to the risk of *greenwashing*, since it is based on data that is published and regulated. This approach makes it possible to address both dimensions of the transition : on one hand, activities already considered sustainable ; on the other hand, investments made by actors in activities transitioning towards greater sustainability.

This framework thus captures the two key dimensions of the transition : first, direct support for activities already aligned with the sustainability criteria of the taxonomy ; and second, support for economic actors in their transformation by financing green investments within sectors that are currently not sustainable.

The main interest lies in taking into account the financial efforts related to the transition via green CAPEX, which form the very core of this regulation. This method complements existing approaches, such as carbon alignment or decarbonization trajectories (SBTi). Indeed, most studies penalize heavily emitting sectors that are nonetheless essential to the economy (metallurgy, energy, transport) without necessarily integrating the financial effort through operating and capital expenditures [1].

Decarbonizing a portfolio frequently leads to the exclusion of key actors in the energy transition, particularly in the energy and utilities sectors. This approach can limit the portfolio's transition capacity, insofar as some low-carbon solutions originate from high-carbon-intensity sectors. It is therefore crucial to differentiate high-carbon emitters that are not aligned with transition objectives from those committed to a trajectory of emission reduction through the adoption of sustainable technologies.

We propose a modeling approach based on modern portfolio theory (Markowitz), to which alignment constraints based on CAPEX and turnover are added. The study focuses on data for fiscal year 2024 on CAC40.

European Taxonomy

The transition to a carbon-neutral European economy requires the massive redirection of private capital towards sustainable activities. The EU Green Taxonomy, launched by the Commission in 2018, serves as an "environmental compass" [2] to guide these investments, forming part of the broader European Green Deal strategy with targets for 2030 and 2050.

The taxonomy evaluates activities, not companies, which allows differentiating sustainable and non-sustainable activities within the same entity. An activity is deemed sustainable if it makes a

1. "Capital spending on physical assets for energy and land-use systems in the net-zero transition between 2021 and 2050 would amount to about $275 trillion, or $9.2 trillion per year on average, an annual increase of as much as $3.5 trillion from today" McKinsey (2022).

2. European Commission, "EU Green Taxonomy : how to use it", Permanent Representation of France to the EU, 13 January 2022, `https://france.representation.ec.europa.eu/informations/taxonomie-verte-mode-demploi-2022-01-13_fr`

substantial contribution to an environmental objective, does not significantly harm other objectives, and respects minimum social safeguards.

The classification distinguishes three types of activities : those directly contributing to an objective, enabling activities that support the former, and transitional activities that improve practices in sectors without viable alternatives.

Finally, the taxonomy is a technical tool updated regularly and applied progressively by sector. Its length and technicality make it a demanding instrument but central for aligning finance with European climate objectives. Until the publication of the delegated regulation in June 2023, only the first two objectives (climate mitigation and adaptation) had fully adopted and applicable technical criteria. Now, the criteria for the four other objectives (water, circular economy, pollution, biodiversity) have also been adopted and have been entering into application progressively since early 2024.

This is characterized in particular by a classification : an activity is said to be aligned if it makes a substantial contribution to an environmental objective, does not significantly harm other objectives, respects minimum social safeguards, and complies with the technical criteria defined by the taxonomy ; an activity is eligible if it appears in the list of sectors covered by the taxonomy, regardless of its level of environmental performance or conformity with technical criteria.

1 Review of recent contributions

Sustainable portfolio construction has become a central concern in finance. In this context, numerous studies have sought to integrate ESG criteria into optimization processes, employing a wide range of approaches, from ESG scores to carbon emissions metrics. This literature review examines the main contributions in this field, highlighting the limitations of existing methods and the growing relevance of standardized regulatory indicators, such as those derived from the EU Green Taxonomy.

The objective of this study is to determine which ESG elements impact investment and are used to construct a portfolio. Initially, publications were selected from several databases such as Scopus, Web of Science, ResearchGate and ScienceDirect in order to answer the research questions.

In the approach of Pedersen et al. (2021), the ESG score reflects both the fundamentals of companies (returns then follow a CAPM adjusted for ESG scores) and investors' preferences for sustainability. Each ESG level corresponds to a maximum Sharpe ratio, defining the ESG-SR frontier that illustrates the trade-off between performance and sustainability. However, studies show a rather strong divergence between ESG scores ; in Berg et al. (2022), the correlations between scores average 0.54 and range from 0.38 to 0.71.

The most common approaches to align a portfolio with the objectives of the Paris Agreement rely on the use of target metrics, of which CO_2 emissions constitute the most frequently employed measure, with various variants (momentum, intensity, etc.). A major problem of these approaches is that they often lead, in an almost systematic manner, to the exclusion or marked underweighting of certain sectors compared to the benchmark index (Cano and Katiyar, 2021; Bolton et al., 2022). Sectors such as energy, utilities or materials, which nonetheless require the most significant capital investments to ensure their transition, are paradoxically those undergoing the largest divestments. The alignment approach is all the more complex given that scope 3 (indirect) emissions are difficult to measure Le Guenedal and Roncalli (2022).

In other studies, such as Hilario-Caballero et al. (2020), a tri-criteria approach is proposed to explicitly integrate investor preferences, notably regarding carbon risk and sustainable investment, by extending the classic Markowitz model via a multi-objective genetic algorithm. This method places greater emphasis on the precise estimation of investor preferences and their impact on portfolio construction.

This paper stands out by introducing an innovative portfolio optimization approach that directly integrates regulatory indicators from the EU Green Taxonomy, namely green CAPEX and green revenue, unlike traditional methods that rely mainly on carbon emissions or inconsistent ESG scores. By anchoring portfolio construction within a robust regulatory framework, it minimizes the risk of greenwashing and enables a reconciliation between financial performance and transition financing. In this sense, the methodology complements existing net-zero approaches : as noted by Roncalli (2025), he already highlighted a key limitation of such approaches, namely that focusing solely on reducing the carbon intensity of a portfolio does not necessarily contribute to increasing the overall greenness of the economy. This method captures both already sustainable activities and ongoing transition efforts, thus offering a more nuanced and pragmatic view of sustainable investing.

2 Selection of the Investment Universe and ESG Data

We consider an optimization over a representative subset of the market. Specifically, we focus on companies listed in the CAC 40 index. The complete list of selected companies is provided in the appendix.

The CAC 40 is composed to include the largest French companies while aiming to ensure some representation of the overall French economy. The advantage of this approach lies in the fact that CAC 40 firms regularly publish standardized financial data. Moreover, their securities are highly liquid, which helps mitigate biases related to liquidity constraints as well as size-related effects such as the small-cap bias. This method is particularly effective for building realistic and implementable portfolios, offering more stable and faster optimization due to the reduced universe size. It also helps limit the risk of overfitting compared to a broader universe. Furthermore, CAC 40 companies generally have greater resources to comply with regulatory and reporting requirements. However, due to constraints related to data availability (drawn from companies' annual reports), the analysis is limited to the CAC 40, which may introduce certain sectoral biases. Moreover, some companies remain uncertain regarding the classification of their activities and the application of the Do No Significant Harm (DNSH) principle.

Regarding eligibility and alignment with the EU Taxonomy, each company is required to disclose this information in its Universal Registration Document. Figure 2.1 and 2.2 illustrate the existence of multiple dimensions of alignment, emphasizing that relying on a single metric is insufficient to fully assess decarbonization efforts and alignment toward a sustainable economy. For instance, TotalEnergies allocates nearly 16% of its capital expenditures (CAPEX) to sustainable activities, whereas only 2% of its revenues are aligned with the taxonomy criteria. Moreover, as we can see, the alignment in terms of capital expenditure is stronger than the alignment in terms of revenues, which already indicates that portfolio alignment with CAPEX targets is easier to achieve than with revenue targets.

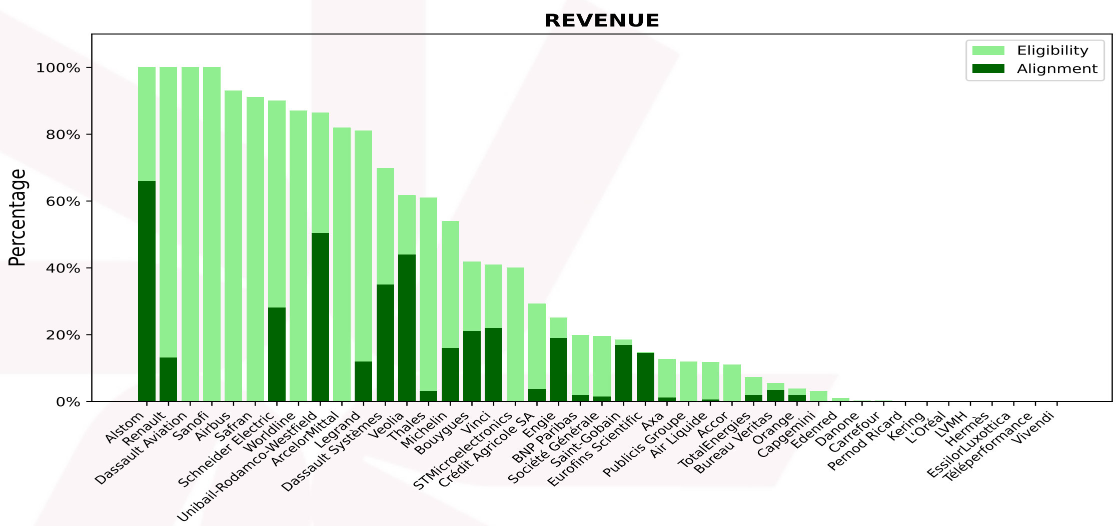

Figure 2.1 Eligibility and Alignment (Revenue) of CAC 40 Companies in 2024 [3]

3. Data based on investment KPIs for insurers and the Green Asset Ratio (GAR) for banks.

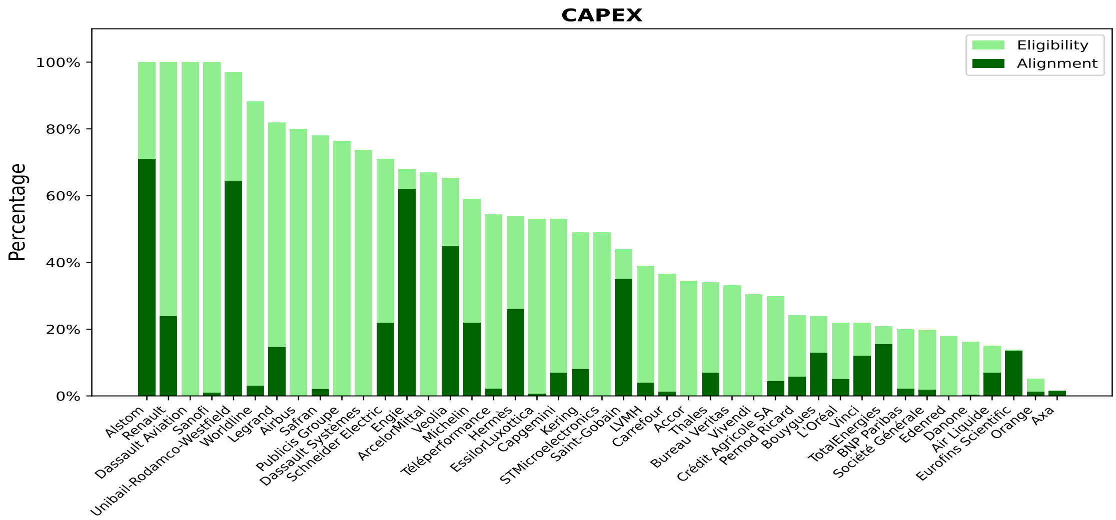

Figure 2.2 Eligibility and Alignment (CAPEX) of CAC 40 Companies in 2024

3 Methodology

In this section, we present the methodological framework used to construct, model, and evaluate investment portfolios. Our approach builds on the foundations of modern portfolio theory, starting with the Markowitz mean-variance optimization, which defines efficient portfolios based on return-risk trade-offs. We then incorporate the Capital Asset Pricing Model (CAPM) to estimate expected returns and derive a structured form of the covariance matrix. Finally, we extend the traditional optimization problem by introducing constraints related to sustainability, using environmental indicators aligned with the EU Taxonomy to assess green investment alignment. To formalize this approach, we first recall the main principles of the Markowitz mean–variance optimization, which provides the theoretical foundation of our model.

3.1 Portfolio Optimization : Theoretical Framework (Markowitz)

Markowitz (1952) formalized portfolio optimization by treating expected return as the objective and return variance as the risk to be minimized. He demonstrated that an efficient portfolio maximizes expected return for a given level of risk (variance). However, Tobin (1958) showed that if a risk-free asset exists, an optimized portfolio will dominate all others. This approach remains a cornerstone of financial theory, despite the emergence of more sophisticated models such as Fama and French (1993) and Black and Litterman (1992).

Let there be an investment universe of n assets. The vector of annualized expected returns is denoted by $\mu \in \mathbb{R}^n$, and the annualized covariance matrix by $\Sigma \in \mathbb{R}^{n \times n}$. Let r_f represent the risk-free rate.

A portfolio is defined by a weight vector $w \in \mathbb{R}^n$ such that $\sum_{i=1}^n w_i = 1$ and $w_i \geq 0$ for all i.

The expected return and variance of the portfolio are :

$$R_p = w^\top \mu, \quad \sigma_p^2 = w^\top \Sigma w, \quad \sigma_p = \sqrt{w^\top \Sigma w}.$$

The minimum variance portfolio is obtained by solving :

$$\min_w \quad w^\top \Sigma w \quad \text{subject to} \quad \sum_{i=1}^n w_i = 1, \quad w_i \geq 0.$$

Its return and risk are :

$$R_{\text{MV}} = w_{\text{MV}}^\top \mu, \quad \sigma_{\text{MV}} = \sqrt{w_{\text{MV}}^\top \Sigma w_{\text{MV}}}.$$

The efficient frontier is obtained by solving, for each target return R_t,

$$\min_{w} \quad w^\top \Sigma w \quad \text{subject to} \quad \sum_{i=1}^{n} w_i = 1, \quad w^\top \mu = R_t, \quad w_i \geq 0.$$

This gives the set of optimal portfolios $\{(R_t, \sigma_p(R_t))\}$, where the efficient frontier corresponds to those with $R_t \geq R_{\text{MV}}$.

The tangency portfolio maximizes the Sharpe ratio :

$$\text{Sharpe}(w) = \frac{w^\top \mu - r_f}{\sqrt{w^\top \Sigma w}},$$

and lies at the tangency point between the capital market line (CML) and the efficient frontier.

Its return and risk are :

$$R_T = w_T^\top \mu, \quad \sigma_T = \sqrt{w_T^\top \Sigma w_T}.$$

The capital market line is given by :

$$R = r_f + \left(\frac{R_T - r_f}{\sigma_T} \right) \sigma, \quad \sigma \geq 0,$$

and represents combinations of the risk-free asset and the tangency portfolio.

3.2 Modeling Expected Returns : The CAPM Approach

The Capital Asset Pricing Model (CAPM) provides a foundational framework for linking expected return to systematic risk. Initially introduced by Treynor (1961), and formalized by Sharpe (1964) and Lintner (1965), the CAPM posits that only systematic (non-diversifiable) risk, measured by the beta coefficient β_i, is rewarded. This model offers several advantages. In particular, it avoids relying on historical or empirical estimates of returns and covariances, which are often highly unstable. It is also distinguished by the parsimony and statistical significance of its parameters, as well as by its strong grounding in asset management practices, with betas being observable measures commonly used in practice.

The beta of asset i is estimated via the linear regression :

$$R_{i,t} - r_f = \alpha_i + \beta_i (R_{m,t} - r_f) + \epsilon_{i,t},$$

with $\mathbb{E}[\epsilon_{i,t}] = 0$ and $\text{Cov}(\epsilon_{i,t}, R_{m,t} - r_f) = 0$.

OLS (Ordinary Least Squares) estimation yields :

$$\widehat{\beta}_i = \frac{\widehat{\text{Cov}}(R_i, R_m)}{\widehat{\text{Var}}(R_m)}, \quad \widehat{\alpha}_i = \overline{R_i - r_f} - \widehat{\beta}_i \, \overline{R_m - r_f}.$$

This is consistent with the CAPM theoretical equation :

$$\mathbb{E}[R_i] = r_f + \beta_i \left(\mathbb{E}[R_m] - r_f \right).$$

The beta β_i measures the sensitivity of asset i to market movements : $\beta_i > 1$ implies higher volatility than the market, $\beta_i < 1$ implies lower volatility, and $\beta_i = 1$ means the asset tracks the market.

Regression diagnostics and robustness To validate OLS estimation, the following diagnostics are recommended :

- Test for statistical significance of α_i.
- Heteroskedasticity tests on residuals $\epsilon_{i,t}$.
- Autocorrelation tests on residuals.

These tests ensure the reliability of OLS assumptions or allow for correction using robust standard errors.

Covariance matrix under single-factor CAPM Under the CAPM, the covariance between two assets i and j is :

$$\mathrm{Cov}(R_i, R_j) = \beta_i \beta_j \,\mathrm{Var}(R_m) + \delta_{ij}\, \sigma^2_{\epsilon,i},$$

where :

- $\mathrm{Var}(R_m)$ is the market variance ;
- $\sigma^2_{\epsilon,i}$ is the idiosyncratic variance of asset i ;
- δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ if $i = j$, 0 otherwise).

This avoids computing all empirical covariances by using betas and market variance.

The matrix Σ is constructed with :

$$\Sigma_{ij} = \beta_i \beta_j \,\mathrm{Var}(R_m) + \begin{cases} \sigma^2_{\epsilon,i}, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases}$$

We replace it with empirical estimates $\widehat{\beta}_i$, $\widehat{\mathrm{Var}}(R_m)$, and $\widehat{\sigma}^2_{\epsilon,i}$.

3.3 Portfolio Evaluation : Diversification, Performance, Environmental Alignment

To ensure that investment portfolios are not only financially sound but also aligned with sustainability goals, it is necessary to introduce a set of evaluation measures across several dimensions. These measures allow investors to assess risk, return efficiency, and environmental objectives in a consistent and quantitative way.

Diversification is measured using the Herfindahl-Hirschman Index (HHI) :

$$\mathrm{HHI} = \sum_{i=1}^{n} w_i^2,$$

where w_i is the weight of asset i. A high HHI indicates concentration ; a low value indicates better diversification.

Performance is assessed via the Sharpe ratio of the tangency portfolio, which captures the efficiency of risk-adjusted returns.

Environmental alignment is measured using green finance indicators from the EU Taxonomy. Two metrics are used :

$$\mathcal{GC}_i = \frac{\text{Green CAPEX}_i}{\text{Total CAPEX}_i}, \quad \mathcal{GT}_i = \frac{\text{Green Revenue}_i}{\text{Total Revenue}_i}.$$

We formulate a constrained quadratic optimization problem to construct portfolios balancing return, risk, and sustainability. For a target return $R \in [\min \mu_i, \max \mu_i]$, we solve :

$$\min_{\boldsymbol{w} \in W} \; \boldsymbol{w}^\top \Sigma \boldsymbol{w} \quad \text{s.t.} \quad \begin{cases} \langle \boldsymbol{w}, \boldsymbol{\mu} \rangle = R \\ \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \\ \langle \boldsymbol{w}, \mathcal{GC} \rangle \geq \tau \quad \text{and/or} \quad \langle \boldsymbol{w}, \mathcal{GT} \rangle \geq \tau \end{cases}$$

where $\boldsymbol{w} = (w_1, \ldots, w_n)^\top$ is the portfolio weight vector, $\boldsymbol{\mu}$ is the vector of expected returns, Σ the covariance matrix, $\mathcal{GC}$ the vector of green CAPEX ratios, $\mathcal{GT}$ the vector of green revenue ratios, and $\tau \in [0, 1]$ is the minimum environmental alignment threshold.

4 ESG-Constrained Optimization : “Fully Invested & Long Only” Strategy

In this section, we study the efficient frontier imposing standard constraints : no short-selling and full investment of the portfolio. Figure 4.1 is built using the aforementioned constraints. Each point on the curve represents the minimum volatility achievable for a given target return, subject to positive weights and a unit-sum constraint.

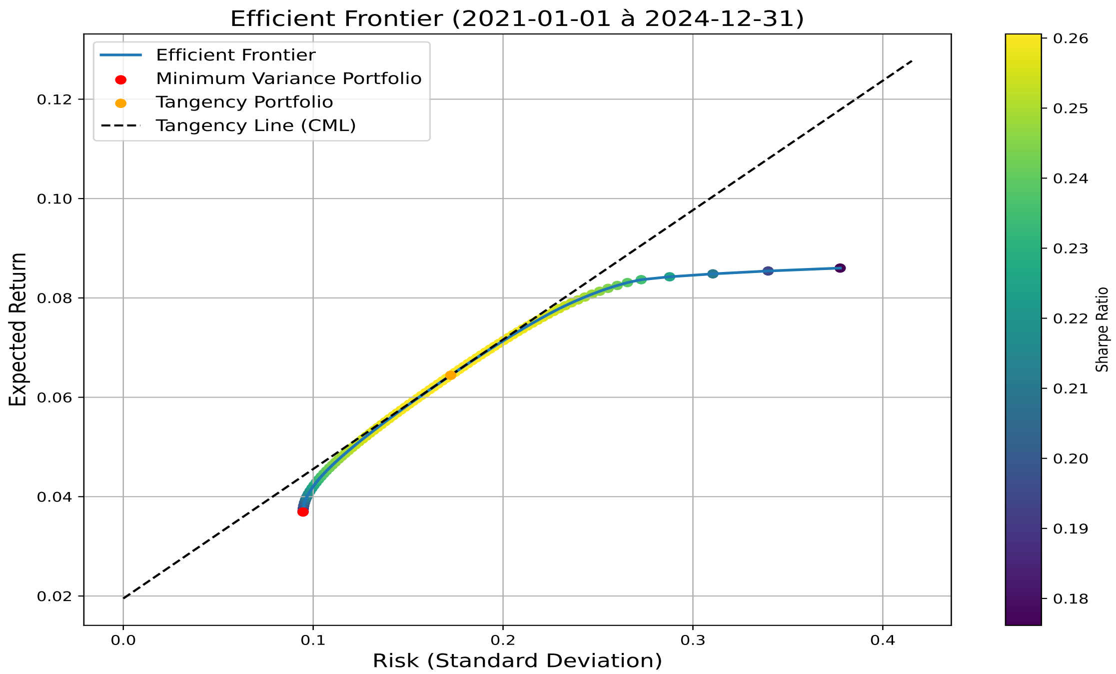

Figure 4.1 Efficient frontier of portfolios constructed from CAC 40 stocks.

Figure 4.2. displays the weights of the tangency portfolio, computed using historical returns of CAC 40 constituents over the sample period. The tangency portfolio includes nearly all stocks from the investment universe, with some assigned very small weights.

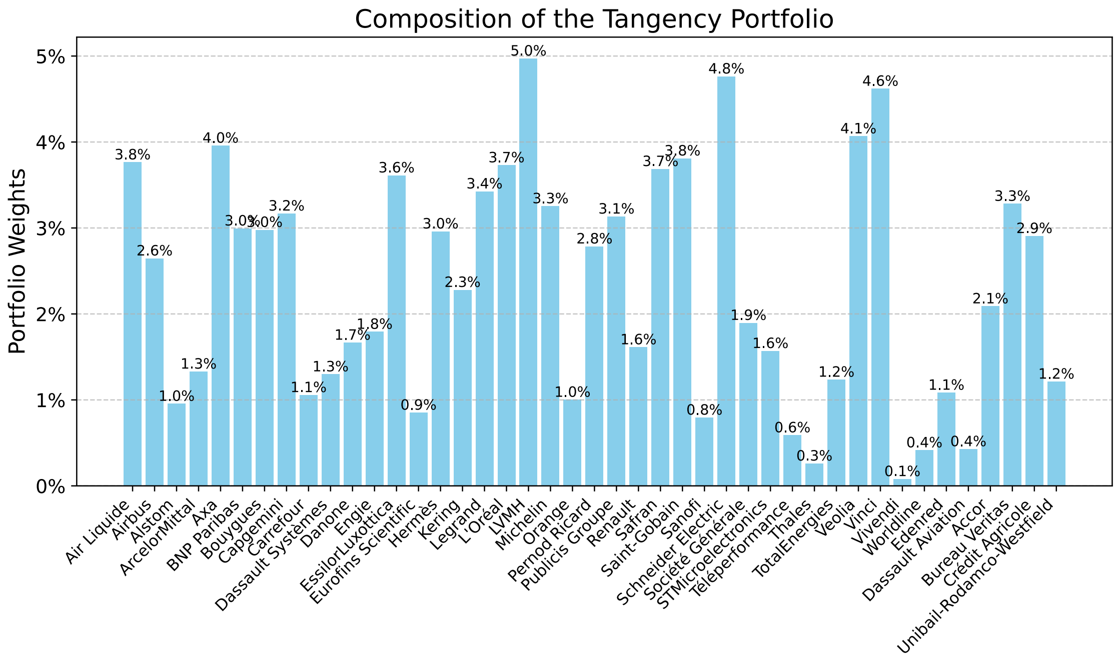

Figure 4.2 Efficient portfolio constructed from CAC 40 stocks

We explore the trade-off between alignment, return and volatility, assessing whether it is possible to achieve meaningful alignment without significantly increasing portfolio volatility. The analysis further investigates the evolution of the tangency portfolio and examines the extent to which green investments can be reconciled within a unified portfolio framework.

4.1 Impact of Green CAPEX Constraints

The green CAPEX constraint acts as a filter toward companies more likely to be resilient to future climate policies (carbon taxes, environmental standards). It potentially reduces the risk of stranded assets. Some studies suggest that firms investing in sustainability tend to perform better financially over the long term (via innovation, energy efficiency, or improved reputation), Poursoleyman et al. (2022).

For all $R \in [\min \mu_i, \max \mu_i]$, we solve the following quadratic problem :

$$\min_{\boldsymbol{w} \in W} \ \boldsymbol{w}^\top \Sigma \boldsymbol{w} \quad \text{s.t.} \quad \begin{cases} \langle \boldsymbol{w}, \boldsymbol{\mu} \rangle = R \\ \sum w_i = 1, \quad w_i \geq 0 \\ \langle \boldsymbol{w}, \mathcal{GC} \rangle \geq \tau \end{cases}$$

where :

- $\boldsymbol{w} = (w_1, \ldots, w_n)^\top$ is the portfolio weight vector,
- $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)^\top$ is the expected returns vector,
- $\Sigma \in \mathbb{R}^{n \times n}$ is the covariance matrix of returns,
- $\mathcal{GC} = (\mathcal{GC}_1, \ldots, \mathcal{GC}_n)^\top$ is the vector of green CAPEX shares according to the taxonomy,
- $\tau \in [0, 1]$ is the minimum desired green CAPEX alignment threshold for the portfolio.

The additional constraint may deteriorate the efficient frontier, leading to higher risk for a given return or vice versa. By restricting allocation to companies with a sufficient share of green CAPEX, high-performing but less sustainable sectors or market leaders may be excluded. This underlines the relevance of a bottom-up approach to track how the portfolio evolves at the company level.

The graph shows that the tangency portfolio already exhibits a certain degree of revenue alignment, around 12%. As the alignment constraint increases, the portfolio gradually concentrates on companies from sectors requiring major investments in the transition, such as energy (Engie), utilities (Veolia), industry (Alstom, Saint-Gobain), and real estate (Unibail-Radamco-Westfield) which are also among the most covered by the green taxonomy. When the alignment requirement exceeds 55%, the investable universe shrinks drastically to just five companies (see Figure 4.3).

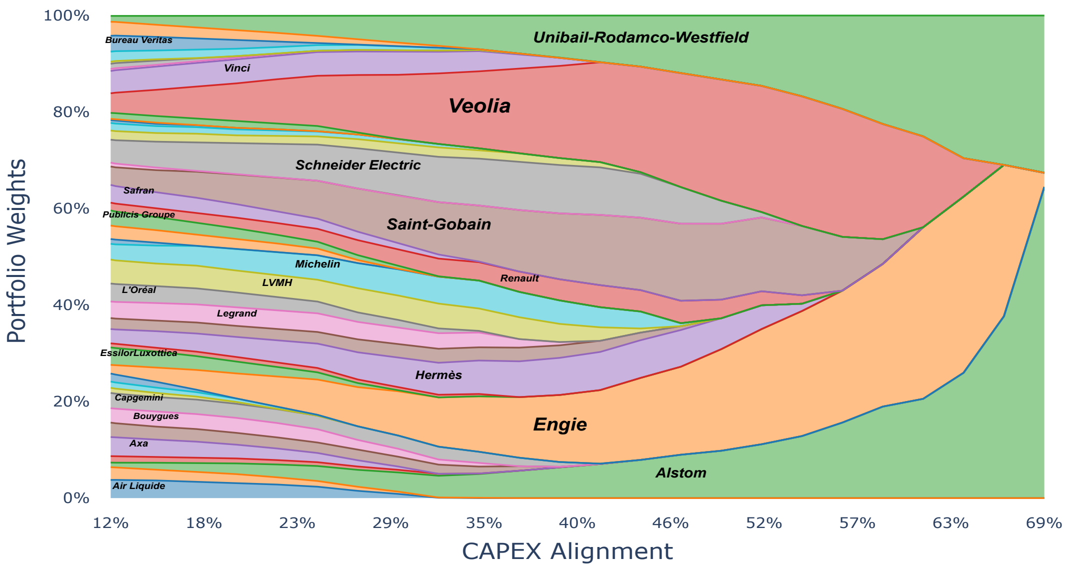

Figure 4.3 Evolution of the tangency portfolio under increasing CAPEX alignment constraint

The key issue lies in balancing transition financing with maintaining desirable financial characteristics, particularly in terms of volatility and target return.

Risk-adjusted returns decline significantly beyond a CAPEX alignment threshold of around 35% (see Figure 4.4). It is possible to increase the alignment ratio by nearly 10 percentage points without significantly impacting diversification (Herfindahl index) or risk-adjusted performance.

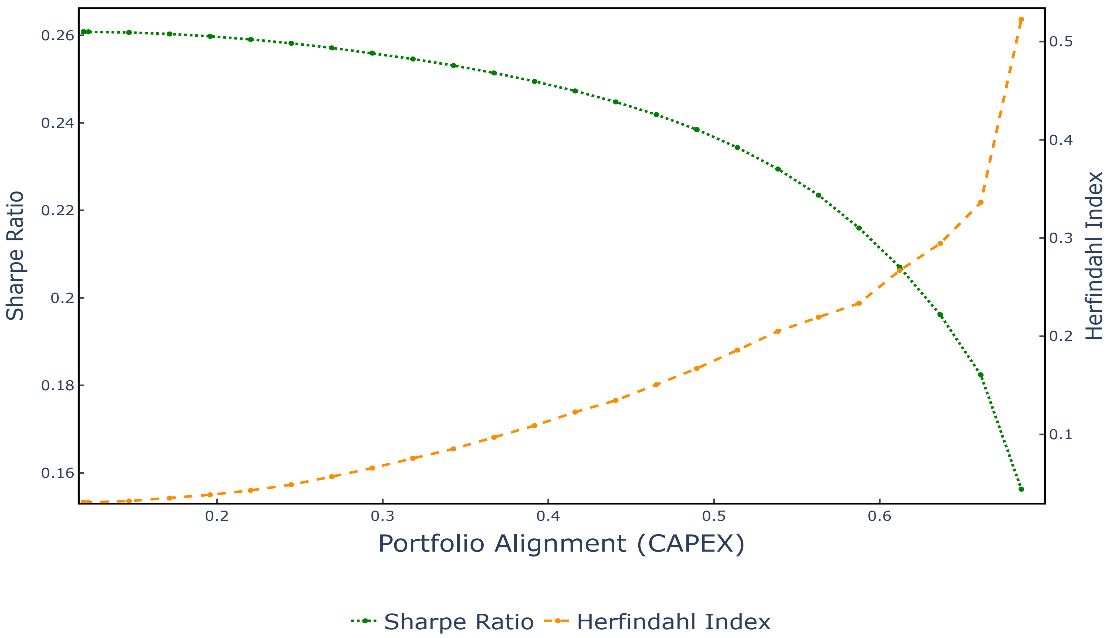

Figure 4.4 Evolution of Sharpe ratio and portfolio concentration under CAPEX alignment constraint

4.2 Impact of Green Revenue Constraints

For all $R \in [\min \mu_i, \max \mu_i]$, we solve the following quadratic problem :

$$\min_{\boldsymbol{w} \in W} \ \boldsymbol{w}^\top \Sigma \boldsymbol{w} \quad \text{s.t.} \quad \begin{cases} \langle \boldsymbol{w}, \boldsymbol{\mu} \rangle = R \\ \sum w_i = 1, \quad w_i \geq 0 \\ \langle \boldsymbol{w}, \mathcal{GT} \rangle \geq \tau \end{cases}$$

where :

- $\boldsymbol{w} = (w_1, \dots, w_n)^\top$ is the portfolio weight vector,
- $\boldsymbol{\mu} = (\mu_1, \dots, \mu_n)^\top$ is the expected returns vector,
- $\Sigma \in \mathbb{R}^{n \times n}$ is the covariance matrix of returns,
- $\mathcal{GT} = (\mathcal{GT}_1, \dots, \mathcal{GT}_n)^\top$ is the vector of green revenue shares according to the taxonomy,
- $\tau \in [0, 1]$ is the minimum desired green revenue alignment threshold for the portfolio.

Figure 4.5 shows a strong portfolio concentration, with an alignment of 60%, the allocation distributed among just three companies : Alstom, Veolia and Unibail-Radamco-Westfield. The challenge is thus to strike a balance between risk reduction and financing the transition.

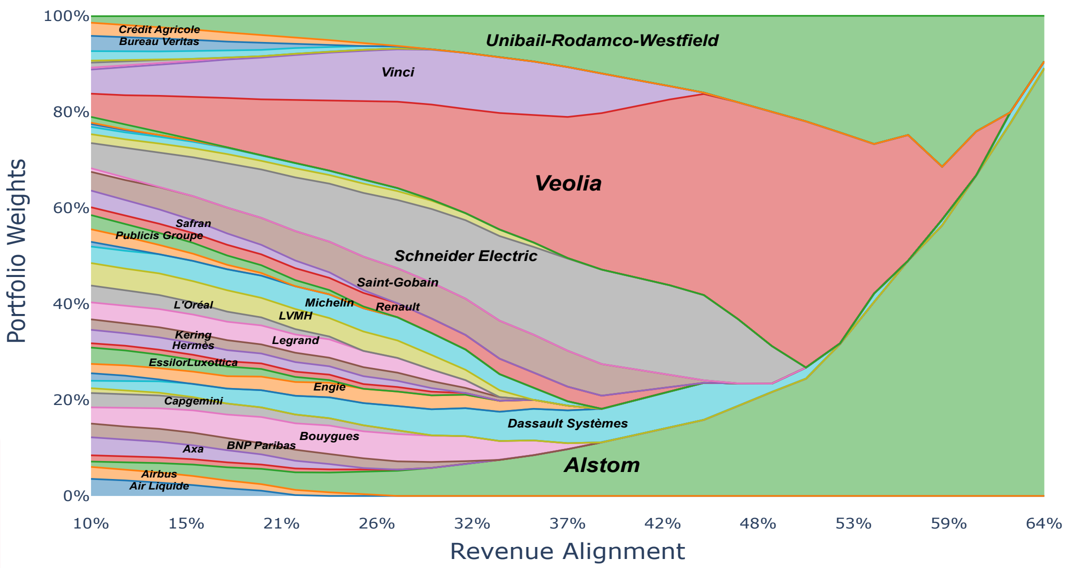

Figure 4.5 Evolution of the tangency portfolio under increasing green revenue alignment constraint

Furthermore, as seen in Figure 4.6, the Herfindahl index increases exponentially with the alignment constraint. However, the risk-adjusted return declines only slightly up to a 25% green revenue alignment.

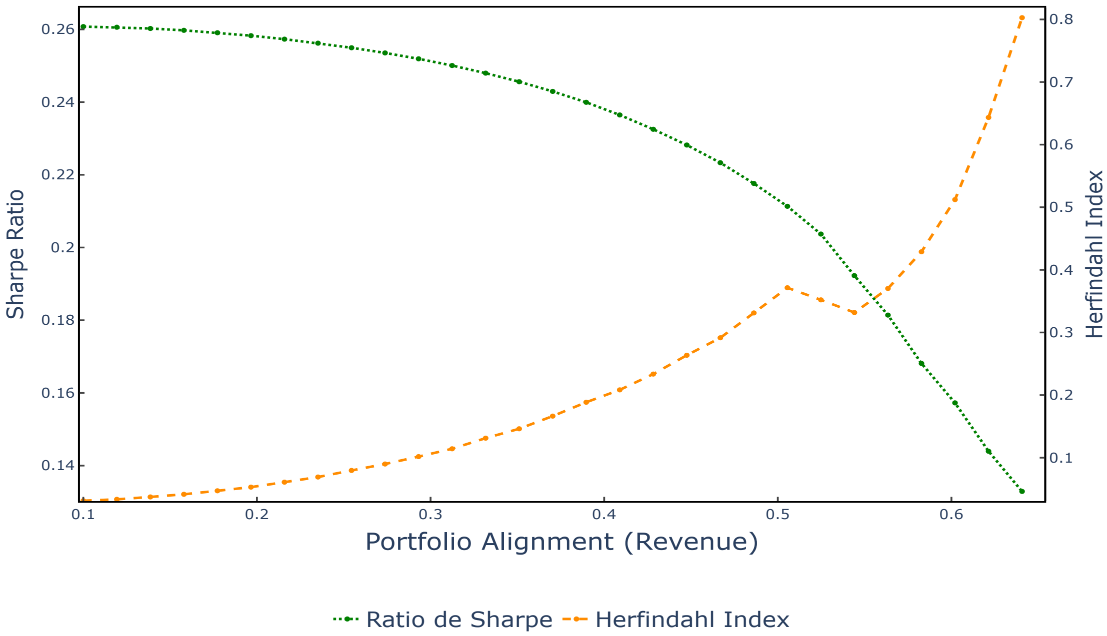

Figure 4.6 Evolution of the Sharpe ratio and portfolio concentration under green revenue alignment constraint

Similarly to the CAPEX alignment case, it is possible to improve the portfolio's alignment with revenue by 10 percentage points without deteriorating the Sharpe ratio. Moreover, certain allocation patterns appear to be consistent between revenue and CAPEX alignment constraints, particularly with notable weights assigned to Veolia and Alstom. However, some divergences emerge : Engie (Utilities sector), previously favored, is no longer prioritized, while takes on a more prominent role in the portfolio composition (Figure 4.5).

4.3 Comparative Analysis : Trade-Off Between Green CAPEX and Revenue

It is possible to improve portfolio alignment without significantly compromising the risk-adjusted return. However, simultaneous alignment on both criteria, green revenues and CAPEX remains a key challenge. Although these two dimensions are correlated, they introduce additional constraints that limit diversification opportunities.

For all $R \in [\min \mu_i, \max \mu_i]$, we solve the following quadratic problem :

$$\min_{\boldsymbol{w} \in W} \ \boldsymbol{w}^\top \Sigma \boldsymbol{w} \quad \text{s.t.} \quad \begin{cases} \langle \boldsymbol{w}, \boldsymbol{\mu} \rangle = R \\ \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \\ \langle \boldsymbol{w}, \mathcal{GC} \rangle \geq \tau \quad \text{and} \quad \langle \boldsymbol{w}, \mathcal{GT} \rangle \geq \tau \end{cases}$$

Figure 4.7 highlights that certain alignment combinations are not feasible within our investment universe, as evidenced by the incomplete coverage of the surface. However, it is still possible to satisfy both constraints simultaneously while maintaining a relatively stable Sharpe ratio. As in previous analyses, excessive alignment on either constraint significantly degrades portfolio performance, with Sharpe ratio losses exceeding 40% for some portfolios.

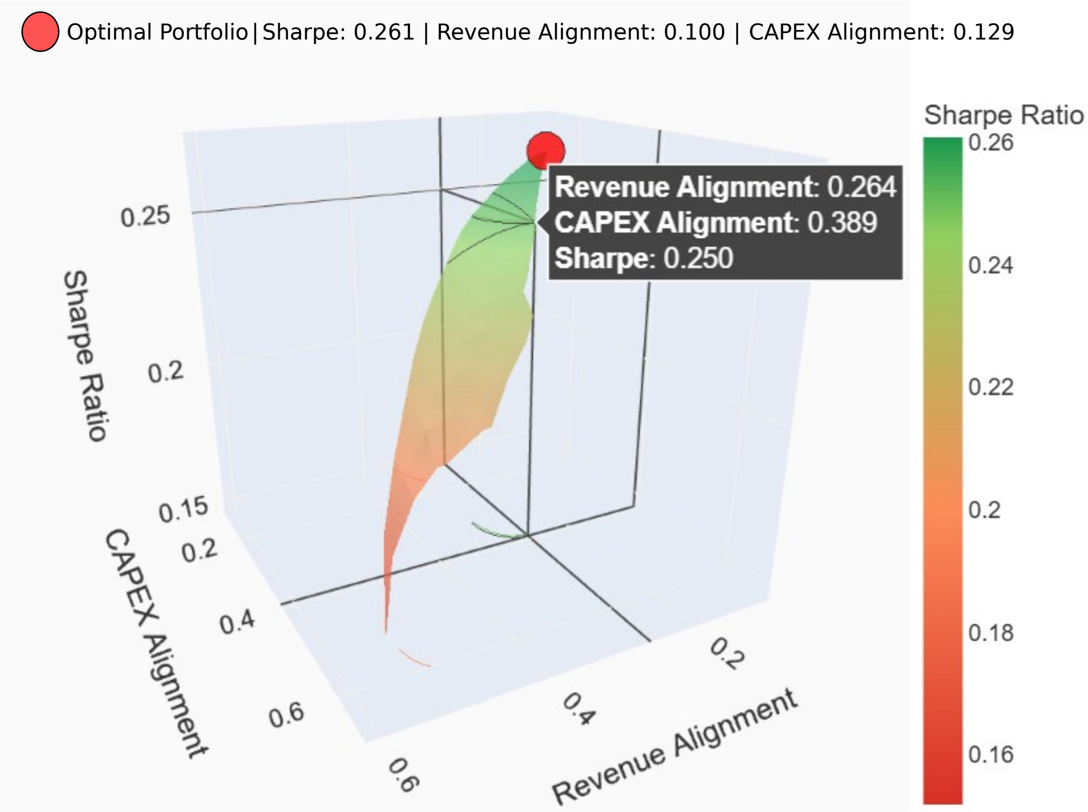

Figure 4.7 Sharpe ratio of the tangency portfolio as a function of revenue and green CAPEX alignment

Figure 4.8 illustrates that the focus remains, albeit less pronounced, when aligning simultaneously with both constraints. It is evident that investments in utilities, such as Veolia, significantly contribute to the portfolio's alignment. Furthermore, as previously observed, there exist opportunities to improve alignment without substantial degradation of diversification or major position changes, as exemplified in Figure 4.8.

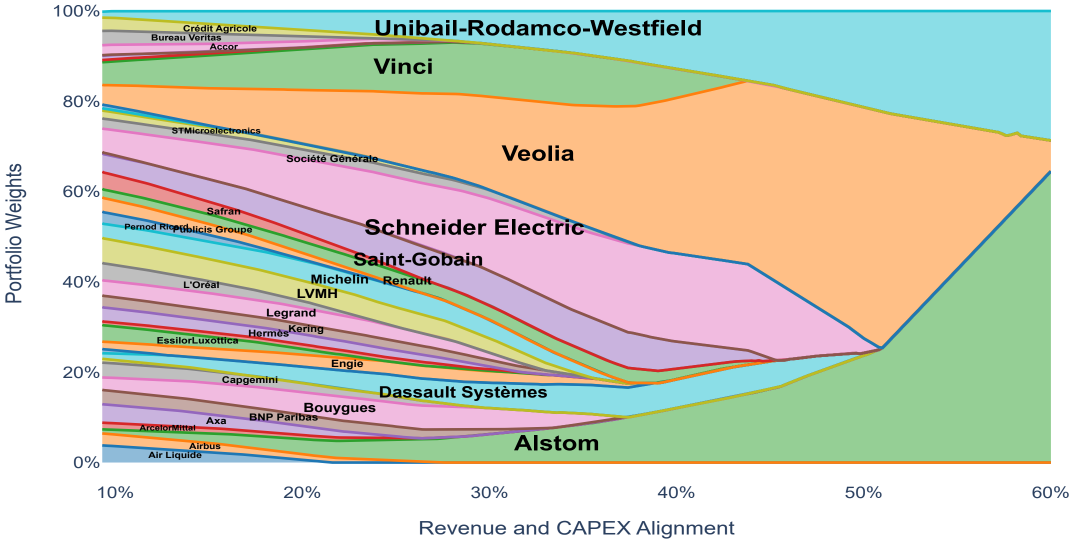

Figure 4.8 Evolution of the tangency portfolio under joint revenue and green CAPEX alignment constraints

5 Analysis and Robustness

5.1 Reconciling the Decarbonization Transition

In this subsection, we analyze whether the portfolio has been decarbonized following the optimization, in terms of carbon intensity Scope 1 and 2 [4], expressed in tons of CO_2 per million dollars of revenue and EVIC (Enterprise Value Including Cash), and then in terms of Scope 3 emissions.

Let $E_{1+2,i}$ denote the Scope 1 and 2 carbon intensity (the same calculation applies to Scope 3) of company i, expressed in tons of CO_2 per million euros of revenue. Portfolio Scope 1 and 2 emissions are calculated as :

$$E_{1+2}^{portfolio} = \sum_{i=1}^{n} \omega_i \times E_{1+2,i}$$

where ω_i represents the weight of company i in the portfolio and $\sum_{i=1}^{n} \omega_i = 1$.

Distinguishing between the different scopes allows us to identify specific dynamics that may be hidden when limiting the analysis to total emissions. This approach is particularly relevant given the structural uncertainties associated with Scope 3 data.

Regarding Scope 1 and 2, we observe an increase in emissions after optimization, regardless of the chosen alignment criterion (CAPEX, revenue, or double alignment). This increase can result from several factors. On the one hand, optimization may have tilted the portfolio toward sectors with higher direct carbon intensity, but deemed more compatible with the transition according to the taxonomy. On the other hand, some companies highly aligned on their future investments may still show high current emissions profiles. Finally, it is possible that optimization favored alignment at the expense of an immediate reduction in direct emissions.

4. Scope 1 : Direct emissions generated by the company and its activities (plants, facilities, warehouses, offices, owned vehicle fleets).
Scope 2 : Indirect emissions related to purchased energy consumption (electricity, steam, heating, cooling).
Scope 3 : All other indirect greenhouse gas (GHG) emissions across the entire value chain, upstream and downstream (transport, travel, product use, etc.).

As for Scope 3, alignment in terms of revenue is accompanied by a slight decrease in Scope 3 emissions, reflecting a modest but real effect of repositioning toward companies more compliant with the taxonomy. In contrast, with alignment based on CAPEX, Scope 3 emissions remain broadly stable, suggesting that projected investments do not yet have a measurable impact on the supply chain or other sources of indirect emissions.

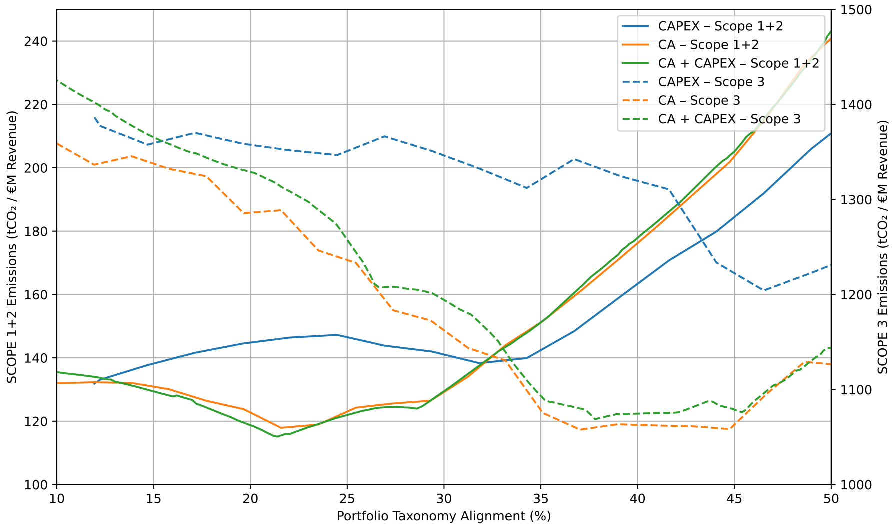

Figure 5.1 Greenhouse gas emissions (tCO_2e per million dollars of revenue)

Carbon Intensity by EVIC (Enterprise Value Including Cash) In addition to the revenue-based analysis, we also study carbon intensity expressed relative to EVIC [5]. This metric has the advantage of taking into account the entire capital structure of the company, attributing emissions proportionally to both shareholders and debt holders. EVIC thus reflects the total value of the company and allows a more equitable allocation of carbon emissions among different financiers.

Portfolio emissions based on EVIC are calculated as :

$$E_{EVIC}^{portfolio} = \sum_{i=1}^{n} \omega_i \times \frac{E_{total,i}}{EVIC_i}$$

where $E_{total,i}$ represents the total emissions of company i (Scope 1+2+3) and $EVIC_i$ its enterprise value including cash.

Emission metrics relative to EVIC lead to the same conclusion : alignment of the portfolio with the European taxonomy is accompanied, in our study of the CAC 40, by an increase in Scope 1 and 2 emissions. By contrast, no clear trend emerges regarding Scope 3 emissions. These conclusions have also been highlighted by analyses of portfolios and indicators aiming to reduce the carbon intensity of investments. In fact, portfolio decarbonization may reduce financing for the most polluting sectors, which nevertheless require significant investments for their transition.

5. EVIC (Enterprise Value Including Cash), as defined by MSCI, is calculated as follows :

$$EVIC = \text{Market Capitalization} + \text{Minority Interests} + \text{Total Debt}.$$

Unlike the traditional Enterprise Value, cash is not deducted from this measure.

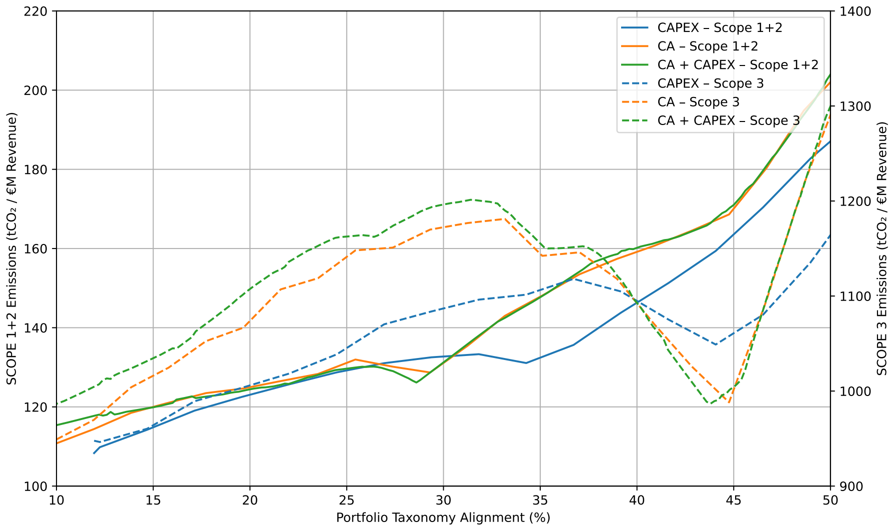

Figure 5.2 Greenhouse gas emissions (tCO_2e per million dollars invested)

5.2 Factor Exposure

The introduction of a constraint in portfolio management may generate biases and unwanted exposures, sometimes unavoidable to satisfy the said constraint. For example, aligning a portfolio with the green taxonomy may indirectly lead to favoring certain management styles (value or growth) or to overweighting small caps.

To analyze and control these effects, the most commonly used method is to rely on factor portfolio data in order to establish the relationship between the manager's portfolio and these reference portfolios. In this framework, let $R_{p,t}$ denote the return of portfolio P at date t, and $R_{f,t}$ the corresponding risk-free rate. The three-factor model of Fama and French (1993) is written, in excess of the risk-free rate :

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_{m,p}\,(R_{m,t} - R_{f,t}) + \beta_{s,p}\,\mathrm{SMB}_t + \beta_{h,p}\,\mathrm{HML}_t + \varepsilon_{p,t},$$

where $R_{m,t}$ is the market return, SMB_t the size factor (Small Minus Big), HML_t the value factor (High Minus Low), and $\mathbb{E}[\varepsilon_{p,t}] = 0$.

The parameters α_p, $\beta_{m,p}$, $\beta_{s,p}$, and $\beta_{h,p}$ are estimated by the ordinary least squares (OLS) method. In practice, performance attribution over the period considered relies on the decomposition of the average excess performance.

We examine the evolution of portfolio exposures to the classical Fama-French factors (market, size, and value style) as a function of their level of alignment. The goal is not to decompose performance, but to understand how alignment influences the risk profile and style of the portfolios.

This approach makes it possible to highlight potential changes in exposure to small caps or to the value style associated with alignment with the green taxonomy, while ensuring that the relationships with factors remain statistically significant.

Thus, based on factor portfolio data published by French, on the European 3-factor portfolios, for the period from January 1, 2021, to December 31, 2024. [6]

Figure 5.3 shows that the tangency portfolio has higher exposure to the HML and SMB factors than the market. This finding is explained by the nature of the CAC 40, a capitalization-weighted index, which overweights large firms and thus reduces its exposure to the SMB factor.

6. Returns include dividends and capital gains and are continuously compounded. Factors are defined as follows : $R_m - R_f$ corresponds to the return of the European market portfolio weighted by capitalization minus the risk-free rate (1-month Euribor). SMB (Small Minus Big) measures the difference in average returns between small and large caps, and HML (High Minus Low) the difference between high and low Book-to-Market ratio stocks, portfolios being constructed by independent 2×3 sorts (size × B/M) at the end of June each year. For more details : `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Developed`

We also observe a systematic increase in exposure to HML (Value) and SMB (Small Caps) factors as the alignment constraint is strengthened, as evidenced by the progression of the associated beta coefficients.

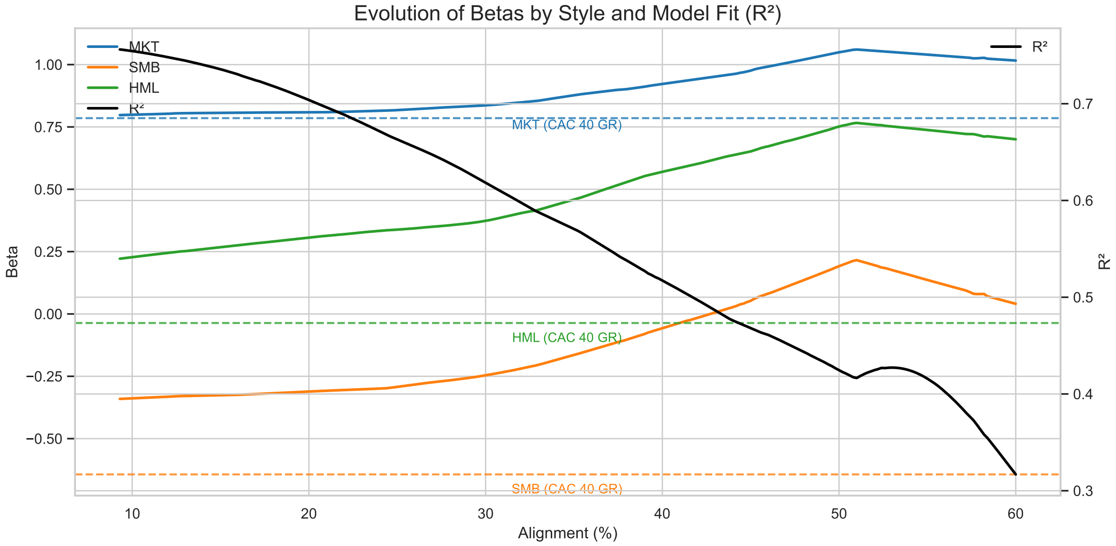

Figure 5.3 Factor exposure according to the alignment constraint

5.3 Forward-Looking Analysis

Since taxonomy data is relatively recent, it is difficult to implement a rigorous backtesting procedure for the methodology. We therefore focus here on the results observed on the portfolio during the period following optimization, i.e., the first half of 2025, in order to check whether the methodology exhibits weaknesses from the outset.

We are aware that this analysis has certain limitations in terms of statistical robustness. Nevertheless, it allows us, as a first step, to assess whether the portfolios were exposed to excessive risk during the study period, as well as to evaluate the impact of the alignment constraint, whether in terms of improvement or deterioration of performance.

To this end, we use stock price data with dividends reinvested, which we compare to an appropriate benchmark index, the CAC 40 Gross Return, corresponding to the CAC 40 index with dividends reinvested.

At first glance, the tangency portfolio shows better performance than the benchmark index, the CAC GR. On the other hand, we observe a gradual decrease in performance as the alignment constraint is tightened (see Figure 5.4).

A particularly interesting point is the small gap observed between the performance of the tangency portfolio and that of the portfolio aligned at 20 %. This finding corroborates the results obtained during optimization : alignment around 20 % only very slightly deteriorates the risk-return trade-off.

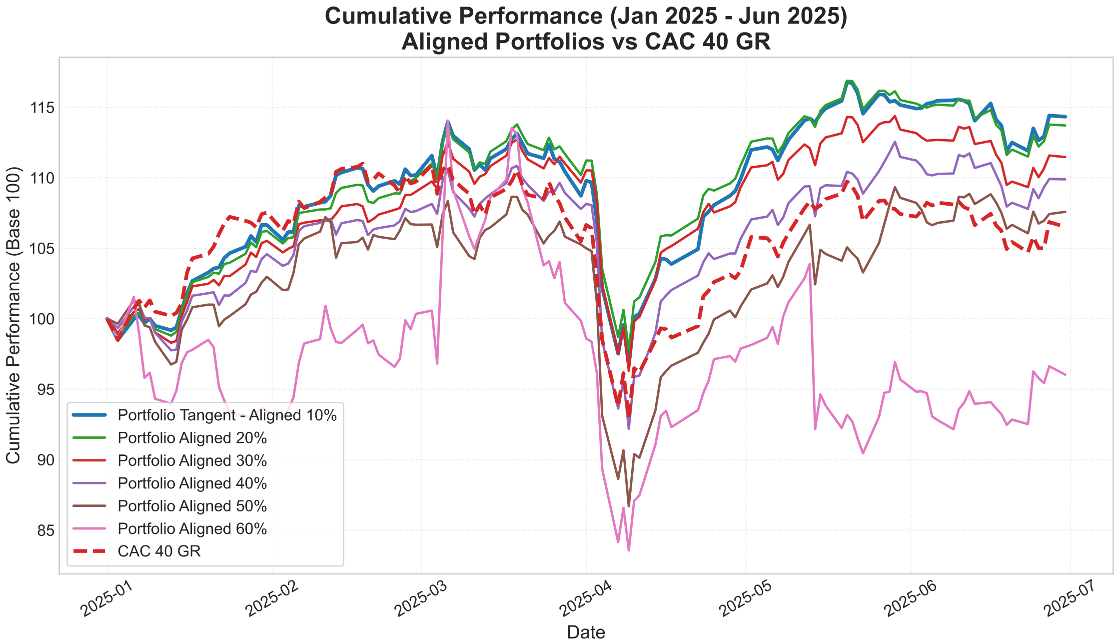

Figure 5.4 Cumulative performance of portfolios

We then analyze the main classical portfolio management metrics :

Jensen's Alpha Jensen's alpha measures the outperformance (or underperformance) of a portfolio relative to its expected return according to the Capital Asset Pricing Model (CAPM). In other words, it measures the portion of portfolio return not explained by systematic risk captured by beta. Formally, if R_p is the annualized portfolio return, R_f the risk-free rate, R_m the annualized market return (benchmark), and β the portfolio beta, then :

$$\alpha = R_p - (R_f + \beta(R_m - R_f))$$

This metric serves to identify the real value added by the manager beyond mere market exposure.

Tracking Error Tracking error measures the volatility of return differences between the portfolio and its benchmark, reflecting the variability of active performance. It is defined as :

$$\text{TE} = \sigma(R_p - R_m) \times \sqrt{N}$$

with σ the standard deviation of return differences over the observed period, and N the number of annualized periods (typically 252 trading days). This metric provides information about the level of specific risk taken by the manager relative to the benchmark.

Information Ratio The information ratio quantifies a portfolio's ability to generate excess return adjusted for specific risk, measured by the volatility of deviations from the benchmark (tracking error). If R_p and R_m denote the annualized returns of the portfolio and the benchmark respectively, and $\text{TE} = \sigma(R_p - R_m)\sqrt{N}$ the annualized tracking error, then :

$$\text{IR} = \frac{R_p - R_m}{\text{TE}}$$

In our case, this ratio measures the quality of the manager's active choices, by relating the performance surplus to the active risk taken.

Treynor Ratio The Treynor ratio evaluates excess performance per unit of systematic risk taken, measured by beta β. It is defined as :

$$\text{Treynor} = \frac{R_p - R_f}{\beta}$$

where R_p is the annualized portfolio return, R_f the risk-free rate, and β the portfolio's sensitivity to the market. In our analysis, this ratio allows us to assess the compensation obtained for each unit of systematic risk borne.

Finally, portfolios with low alignment (10 % to 30 %) exhibit annualized performance well above the benchmark, with a high Sharpe ratio (> 1.2) and controlled volatility ($\sim 17\,\%$). Beyond 40 % alignment, performance deteriorates rapidly while volatility and tracking error increase significantly. At 60 %, performance becomes negative, the Sharpe ratio turns negative, and beta exceeds 1, indicating greater market exposure.
The performance cost of alignment is therefore small up to 30 %, moderate at 40 %, and prohibitive beyond that.

Portfolio	**Ann. Perf. (%)**	**Ann. Vol. (%)**	**Beta**	**Alpha (%)**	**Sharpe**	**Info**	**Treynor**	**Track. Err. (%)**
Tangency – Aligned at 10 %	31.29	17.66	0.90	18.83	1.66	3.32	0.33	5.30
Aligned at 20 %	29.84	17.14	0.85	17.96	1.62	2.39	0.33	6.76
Aligned at 30 %	24.69	17.61	0.83	13.03	1.29	1.27	0.27	8.68
Aligned at 40 %	21.12	19.80	0.87	8.96	0.97	0.66	0.22	11.26
Aligned at 50 %	16.02	23.90	0.96	2.79	0.59	0.15	0.15	15.52
Aligned at 60 %	−7.90	37.64	1.22	−24.12	−0.26	−0.72	−0.08	30.05
CAC GR	13.67	18.91	1.00	–	0.62	–	0.12	–

Table 5.1 Annualized performance metrics of portfolios and the benchmark (Period : 01/01/2025 to 30/06/2025)

Conclusion

This study highlights the relevance of incorporating regulatory indicators such as green CAPEX and green revenues into the construction of financial portfolios. Building on Markowitz's theoretical framework and using an ESG-constrained optimization model, we demonstrate that it is possible to construct portfolios that are both efficient and partially aligned with the European Union's sustainability objectives.

For comparison, a study based on a sample of Article 9 funds by Jespersen et al. (2024) finds that while 80% of Article 9 funds report having sustainable investments, only 40% disclose any taxonomy alignment. On average, these funds report 14% of their portfolios as taxonomy-aligned, compared to 94% labeled as sustainable.

Beyond certain thresholds, ESG constraints lead to increased sectoral concentration and a notable decline in risk-adjusted performance. The joint CAPEX/revenue approach, while conceptually robust, exacerbates these tensions and reveals the limitations of the investment universe used.

The robustness analysis highlights a clear trade-off : alignment with the EU taxonomy improves the portfolio's sustainability profile but comes at the cost of higher Scope 1–2 emissions, stronger style tilts (small caps and value), and increasing performance penalties as alignment intensifies. Moderate alignment levels (around 20–30%) appear to offer a "sweet spot," preserving most of the risk-return efficiency while providing meaningful exposure to green activities.

However, this approach has several limitations. On the one hand, the analysis is restricted to the CAC 40 index, which may introduce sector bias and limit the generalizability of the results to other indices or markets. On the other hand, the data used covers only a single year (2024), which prevents a dynamic or robust assessment of performance across different economic cycles.

Moreover, the taxonomy indicators considered in this study focus exclusively on environmental objectives. While the EU taxonomy is expected to gradually integrate social and governance dimensions, current alignment assessments do not yet reflect the full scope of sustainable development criteria. In addition, companies' self-reported taxonomy data still present significant methodological and practical flaws, including inconsistent interpretation of technical criteria, incomplete disclosure, and delays in compliance. These issues may impact the reliability and comparability of alignment measures across firms.

Ultimately, this work offers a practical contribution to sustainable finance by showing that the progressive integration of alignment criteria can be achieved without compromising core portfolio management principles. It paves the way for future developments, including applications to other indices, time periods, or modeling approaches (such as machine learning or Bayesian methods).

References

Berg, F., J. F. Kölbel, and R. Rigobon (2022). Aggregate confusion : The divergence of esg ratings. *Review of Finance 26*(6), 1315–1344.

Black, F. and R. Litterman (1992). Global portfolio optimization. *Financial Analysts Journal 48*(5), 28–43.

Bolton, P., M. Kacperczyk, and F. Samama (2022). Net-zero carbon portfolio alignment. *Financial Analysts Journal 78*(2), 19–33.

Cano, G. and S. Katiyar (2021, December). Net-zero alignment : Portfolio construction approaches for investors. Research insights, MSCI ESG Research LLC.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Hilario-Caballero, A., A. Garcia-Bernabeu, J. V. Salcedo, and M. Vercher (2020). Tri-criterion model for constructing low-carbon mutual fund portfolios : A preference-based multi-objective genetic algorithm approach. *International Journal of Environmental Research and Public Health 17*(17).

Jespersen, K., A. Almgren, and T. S. Pehrsson (2024). The EU Taxonomy : Challenges and opportunities in sustainable finance. Nordic ESG Lab, Copenhagen Business School.

Le Guenedal, T. and T. Roncalli (2022). Portfolio construction with climate risk measures. SSRN Working Paper.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics 47*(1), 13–37.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance 7*(1), 77–91.

McKinsey (2022). The net-zero transition : What it would cost, what it could bring.

Nexialog (2024). Analyse des rapports Art. 29 LEC 2023 et 2024 : Benchmark des pratiques de durabilité des 23 plus grands assureurs vie.

Pedersen, L. H., S. Fitzgibbons, and L. Pomorski (2021). Responsible investing : The esg-efficient frontier. *Journal of Financial Economics 142*(2), 572–597.

Poursoleyman, E., G. Mansourfar, S. Homayoun, and Z. Rezaee (2022). Business sustainability performance and corporate financial performance : The mediating role of optimal investment. *Managerial Finance 48*(2), 348–369.

Roncalli, T. (2025). Handbook of sustainable finance. Lecture notes, University Paris-Saclay, ENSAE and Sorbonne University.

Sharpe, W. F. (1964). Capital asset prices : A theory of market equilibrium under conditions of risk. *The Journal of Finance 19*(3), 425–442.

Tobin, J. (1958). Liquidity preference as behavior towards risk. *The Review of Economic Studies 25*(2), 65–86.

Treynor, J. L. (1961). Market value, time, and risk.

A CAPM Estimates and Diagnostic Tests for CAC 40 Stocks

Table A.1 Summary of CAPM Regression Results and Diagnostic Statistics for CAC 40 Constituents

Data extraction period : 2021-01-01 to 2024-12-31

Asset	Beta(SE)[p-value]	R^2	DW	Ljung-Box p	JB p	BP p
Air Liquide	0.838(0.062)[0.000]	0.467	2.321	0.038	0.000	0.135
Airbus	1.068(0.089)[0.000]	0.412	2.253	0.226	0.000	0.204
Alstom	1.292(0.172)[0.000]	0.215	1.747	0.415	0.000	0.539
ArcelorMittal	1.168(0.127)[0.000]	0.292	2.177	0.161	0.045	0.927
Axa	1.003(0.068)[0.000]	0.510	1.926	0.036	0.000	0.099
BNP Paribas	1.289(0.092)[0.000]	0.489	1.749	0.169	0.000	0.003
Bouygues	0.739(0.068)[0.000]	0.366	1.621	0.025	0.000	0.380
Capgemini	1.204(0.087)[0.000]	0.485	2.251	0.177	0.000	0.822
Carrefour	0.604(0.102)[0.000]	0.144	2.178	0.223	0.000	0.622
Dassault Systèmes	0.824(0.108)[0.000]	0.220	2.004	0.616	0.000	0.104
Danone	0.403(0.068)[0.000]	0.147	2.182	0.566	0.003	0.000
Engie	0.852(0.095)[0.000]	0.279	2.059	0.916	0.000	0.000
EssilorLuxottica	0.987(0.072)[0.000]	0.477	2.137	0.733	0.414	0.685
Eurofins Scientific	0.784(0.130)[0.000]	0.150	2.096	0.587	0.000	0.202
Hermès	1.299(0.093)[0.000]	0.488	1.904	0.066	0.000	0.174
Kering	1.485(0.110)[0.000]	0.468	2.198	0.344	0.000	0.691
Legrand	0.898(0.070)[0.000]	0.443	2.216	0.042	0.006	0.284
L'Oréal	0.991(0.071)[0.000]	0.488	2.019	0.535	0.000	0.242
LVMH	1.477(0.076)[0.000]	0.648	2.018	0.059	0.000	0.473
Michelin	0.951(0.075)[0.000]	0.438	1.980	0.639	0.000	0.543
Orange	0.264(0.072)[0.000]	0.062	1.997	0.152	0.000	0.412
Pernod Ricard	0.817(0.075)[0.000]	0.364	2.071	0.384	0.178	0.948
Publicis Groupe	1.067(0.082)[0.000]	0.453	2.238	0.384	0.000	0.086
Renault	1.498(0.131)[0.000]	0.388	1.852	0.441	0.016	0.005
Safran	1.082(0.075)[0.000]	0.501	2.005	0.479	0.187	0.809
Saint-Gobain	1.343(0.083)[0.000]	0.557	1.862	0.021	0.022	0.820
Sanofi	0.369(0.093)[0.000]	0.072	2.090	0.766	0.000	0.772
Schneider Electric	1.211(0.068)[0.000]	0.603	2.043	0.579	0.567	0.090
Société Générale	1.564(0.123)[0.000]	0.440	1.673	0.120	0.000	0.000
STMicroelectronics	1.248(0.120)[0.000]	0.343	1.694	0.087	0.000	0.551
Téléperformance	0.808(0.173)[0.000]	0.096	2.104	0.657	0.000	0.169
Thales	0.206(0.111)[0.064]	0.017	2.192	0.724	0.000	0.000
TotalEnergies	0.520(0.090)[0.000]	0.140	1.895	0.465	0.034	0.010
Veolia	1.215(0.076)[0.000]	0.553	2.350	0.028	0.005	0.000
Vinci	1.025(0.062)[0.000]	0.572	1.993	0.348	0.024	0.249
Vivendi	0.541(0.361)[0.135]	0.011	2.375	0.114	0.000	0.846
Worldline	1.294(0.228)[0.000]	0.135	2.285	0.171	0.000	0.917
Edenred	0.663(0.106)[0.000]	0.160	2.201	0.281	0.000	0.330
Dassault Aviation	0.314(0.111)[0.005]	0.037	2.132	0.891	0.008	0.225
Accor	1.180(0.103)[0.000]	0.387	2.043	0.997	0.000	0.900
Bureau Veritas	0.819(0.068)[0.000]	0.414	2.068	0.942	0.000	0.336
Crédit Agricole	1.111(0.087)[0.000]	0.444	1.809	0.117	0.044	0.027
Unibail-Rodamco-Westfield	1.200(0.136)[0.000]	0.273	2.327	0.013	0.000	0.945

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